Study group on Pan's works

Locally analytic completed cohomology
Maison du Kleebach (Munster, France), November 21-25 2022

The goal of the study group is to understand as much as possible the papers [Pan22a] and [Pan22b]. More precisely, we will study the locally analytic vectors of completed cohomology using new techniques from p-adic Hodge theory such as geometric Sen theory [RC22a], or the construction of the Fontaine operator of [Pan22b, §6]. As an application, Pan obtains a geometric proof of a theorem of Emerton regarding the classicality of de Rham Galois representations appearing in completed cohomology, see [Pan22b, Theorem 7.1.2].

Talk 1: Locally analytic representations

In this talk we review the basics on locally analytic representation theory of p-adic Lie groups. The lecturer can follow Pan's classical approach of the first sections of [Pan22a, Pan22b], and use some results of [RJRC21] as blackbox. In future lectures we will need the notion of derived locally analytic vectors, and the comparison between completed, locally analytic and Lie algebra cohomology.

Talk 2: Geometric Sen theory

This is one of the main tools in p-adic Hodge theory introduced by Pan [Pan22a], and generalized in [RC22a]. The goal of this talk is to set up the formalism of Sen theory, define the geometric Sen operator, and show how Sen theory helps to compute group cohomology and apply this to the completed structural sheaf of a torsor of rigid spaces. The lecturer can follow the set up of [Pan22a], and read the approach of [RC22a] to have a more conceptual understanding of the theory.

Talk 3: Equivariant sheaves over $\mathbb{P}^1$ and the π_{HT}-period map

The goal of this talk is to describe explicitly the pullbacks by π_{HT} of GL_2-equivariant sheaves over $\mathbb{P}^1$. As a particular case, the lecturer should explain how the Hodge-Tate weight of classical modular sheaves are interpreted from $\mathbb{P}^1$, how to recover the Faltings extension and as a consequence how to compute the Sen operators of modular curves, see [Pan22a, Theorem 4.2.2] and [RC22b, Theorem 4.3.1 and 4.3.1].

Talk 4: Completed cohomology and locally analytic vectors

In this talk we use geometric Sen theory to the case of modular curves, obtaining the description of completed cohomology in terms of the sheaf ^{la} as in [Pan22a, Theorem 4.4.6] or [RC22b, Theorem 4.4.16]. It would be worth it to explain how the arithmetic Sen operator is related to the Hodge cocharacter, see [RC22b, Theorem 4.4.18]. If time permits the lecturer could explain the relation between $\mathcal{D}$-modules over the flag variety and the isotypic part for the action of a Borel in the completed cohomology, see [Pan22a, §5] or [RC22b, §5]. Another option is to explain how the computation of the Sen operator helps to deduce the classical Hodge-Tate decomposition of étale cohomology in terms of coherent cohomology, see [RC22b, Theorem 4.3.6].

Talk 5: The Fontaine operator

In this lecture we explain Fontaine's generalization of Sen theory to B_{dR}, as covered in §6.1 of [Pan22b] (it is of course a good idea for the speaker to also look at the original paper of Fontaine, but he/she should make sure to use the setup and notations from [Pan22b]). Then explain how this formalism applies to the infinite level modular curves (beginning of §6.2, before the statement of Theorem 6.2.6).

Talk 6: The operators d and $\bar{d}$

The goal of this lecture is to introduce the operators d and $\bar{d}$ over the $(0,0)$-weight of the locally analytic sheaf ^{la}, see [Pan22b, §4.1 and 4.2]. The lecturer should also introduce the intertwining operator of §4.3. Finish by stating Theorem 6.2.6 comparing the intertwinning operator to Fontaine's operator on completed cohomology. (If time permits the speaker could explain the structure of the $\mathfrak{n}$-invariants of $\ker I_k^1$ as in §4.4, see [Pan22b, Theorem 4.4.15].)

Talk 7: Proof of Theorem 6.2.6

The speaker should explain the the proof of Theorem 6.2.6 of [Pan22b] (§6.2-§6.5). This is probably the hardest (and most novel) part of the paper. It would be amazing if some arguments or methods in the proof can be done in a more conceptual way, but it is likely that this will be discussed after the talk.

Talk 8: Locally analytic vectors of Ω_∞

The goal of this lecture is to cover §5.2 and §5.3 of [Pan22b]. We want to show that the locally analytic vectors of the Drinfeld and Lubin-Tate sides are the same at infinite level. The speaker could try to use the abstract geometric Sen theory to avoid reproving some theorems of [Pan22b], and focus in the relevant details.

Talk 9: I_k on $\mathbb{P}^1(\mathbb{Q}_p)$ and Ω

In this talk the speaker should describe the behavior of I_k on overconvergent locus of $\mathbb{P}^1(\mathbb{Q}_p)$ and on Ω, covering §5.1 and §5.4 of [Pan22b]. Then the lecturer should use the previous descriptions to explain the spectral decompositions of Theorem 5.5.4 of [Pan22b].

Talk 10: The Fontaine operator and completed cohomology

Finally, the last speaker should explain how the Fontaine operator of completed cohomology is related with the local Fontaine operator of the sheaf $\mathbb{B}_{dR}^{la}$, and if possible give a sketch of the proof of Theorem 7.1.2 of [Pan22b]. If time permits, the speaker could explain the relation of Pan's method with the p-adic local Langlands correspondence of §7.3.

References

[Pan22a] Lue Pan. On locally analytic vectors of the completed cohomology of modular curves. *Forum of Mathematics, Pi*, 10:e7, 2022.

[Pan22b] Lue Pan. On locally analytic vectors of the completed cohomology of modular curves ii, 2022.

[RC22a] J. E. Rodríguez Camargo. Geometric sen theory over rigid analytic spaces. `https://arxiv.org/abs/2205.02016`, 2022.

[RC22b] J. E. Rodríguez Camargo. Locally analytic completed cohomology. `https://arxiv.org/abs/2209.01057`, 2022.

[RJRC21] Joaquín Rodrigues Jacinto and Juan Esteban Rodríguez Camargo. Solid locally analytic representations of p-adic Lie groups. `https://arxiv.org/abs/2110.11916`, 2021.